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The factors effecting teaching linear algebra

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Abstract

Linear algebra offers, with calculus, the two main mathematical fields taught in universities. It has an active area for research works in mathematics education in several countries. Linear algebra today serves large and very diverse students. How much emphasis should be placed on proofs? How abstract should the setting be? What contribution can be made by technology? How do students learn linear algebra, and what teaching methods are more effective? In this paper, it is discussed the factors effecting linear algebra by the help of the mathematics education researches.

The main issues it will be addressed in this paper concern:
Formalism
The Central Curriculum
Learning Profile of Students
Teaching Strategies
Using Technology in Linear Algebra

Keywords: formalism; curriculum; teaching strategies; technology.

1. **Introduction**

In most universities, science-orientated curricula contain the courses, namely, calculus and linear algebra. Mathematics education research first developed works on calculus, but in the past 20 years, many studies has been carried out about the teaching and learning of linear algebra. One can distinguish roughly two main traditions in the teaching of linear algebra: one focuses on the study of formal vector spaces while the other proposes a more analytical approach based on the study of R^n and matrix calculus (Dorier, 1995).

There are deeper issues that deserve some thought. Linear algebra today serves large and very diverse students. How much emphasis should be placed on proofs? How abstract should the setting be? What contribution can be made by technology? How do students learn linear algebra, and what teaching methods are more effective?

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(Carlson, Johnson, Lay and Porter, 1993). Concern about these questions has produced a healthy interest in linear algebra education.

It can be easy to get into a routine when we have taught a course a few times. The material to be followed, the instruments of the course and grades, all become familiar (Carlson, 1993). The teacher has little hesitation about what must be done each day at the course. On that situation, every teacher knows how to teach linear algebra.

The availability of computers and software has forced mathematicians to rethink the way they are teaching mathematics. When an algorithm can be performed quickly and satisfactorily by a computer, one has to ask `what is it that a student really needs to learn?' (Herrero, 2000).

The aim of this text is to discus the factors effecting linear algebra by the help of the mathematics education researches.

2. Formalism

Formalism is a philosophical approach seeing at mathematicians in the middle of twentieth century. According to a modern formalist, mathematics is a play made by interferences. A formalist defines mathematics as an exact proof science. There is only one way of making mathematics that is proof. When you reach to the conclusions of the proof mathematics is finish. The level of success is evaluated with the number of proofs. Whether others understand it is not a mathematical question (Davis and Hersh, 1996).

Students' difficulties with the formal aspect of linear algebra are not just a general problem with formalism but mostly a difficulty of understanding the specific use of formalism within linear algebra. Some diagnostic studies were made on the question what the authors termed 'the obstacle of formalism' (Dorier, Robert, Robinet, Rogalski, 2000a-b-c; Dorier, 1998; Robert 2000 and Rogalski, 1996). The main point of these studies which Dorier called 'meta level activities' is that linear algebra students have to be used their previous knowledge and have to be related with new formal concepts in linear algebra. In this point somebody may think that in order to teach linear algebra, one way would be give up showing the formal theory of linear algebra. But, it is vital that students starting mathematics departments of university need to have some idea about the formal concepts and axiomatic algebraic structures of which linear algebra is one of the most important.

3. The Central Curriculum

Linear algebra is taught to different students from different faculties. These faculties expect linear algebra course to play its proper roles in their curriculum; Education faculties expect it to contributing the abstract thinking of their teacher candidates. Engineering faculties expect it to make emphasis on physical science applications and computer experiences. Maybe some programs intend to teach linear algebra at the secondary level.

The question that 'what can be the basic linear algebra curriculum?' is an interesting research subject for the persons studying on linear algebra education. First, in 1990, the Linear Algebra Curriculum Study Group recommended a basic curriculum for a first linear algebra course (Carlson at all, 1993). There were linear algebra teachers from diverse universities in this group. They interviewed with a variety of experts who teach linear algebra, and they developed their studies after much discussion. Their study was not published in 1993, and its effect was felt quickly via special educational meetings. Also, in several countries, many linear algebra textbook authors influenced this curriculum reform (Bretscher, 1997; Cullen, 1996; Larson and Edvards, 1996).

In last decade, software programs (matlab, mathematica, Derive, Linalg) have being used effectively at teaching linear algebra. In 1999, another group from Park City Mathematics Institute tried to update linear algebra curriculum (Dorier, 2000). They carefully investigated different faculty program in which they give linear algebra to their students and talked many linear algebra instructors about how linear algebra curriculum is actually being used today. At the end of the search, they explained that there is no unique linear algebra curriculum would serve the needs of different approaches.

There are also some new interesting approaches to the linear algebra curriculum; Axler (1995) gives an elegant method of eigenvalues and eigenvectors in which he doesn't use determinants. Another related discussion about this subject, Worter and Meyers (1998) present a simple algorithm of eigen values and eigenvectors without using determinants. Pedagogical recommendations regarding the subject give us a useful perspective (Dubinsky, 1997; Hare, 1997 and Harel, 1997).

Finally, what we can say about linear algebra curriculum that every faculty gives in its particular program? Every faculty or department should decide which curriculum model makes sense at their program and which topics they want their student will learn.

4. Learning Profile of Students

Understanding of how students learn what mathematics concepts they know and how they learned them have an important effect on the quality of education. Several studies in the context of linear algebra have demonstrated that students don't generally have a rich conceptual understanding of vector spaces and linear transformations (Dorier, 2000 and Carlson, Johnson, Lay, Porter, Watkins, 1997). I observed that many students have difficulties to learn a vector space generated from a vector set and a linear transformation obtained from the images of base's vectors. So it seems clear that doing a better job of telling and showing as a teacher may not significantly improve students' learning of such difficult topics.

There is not a great deal of published literature on how students understand and learn linear algebra (Dorier, 2000). Harel (1997) has been studying some aspects of this subject for several years, and he gives some suggestions in his articles for linear algebra teachers. Carlson (1993) presents an interesting hypothesis about the special difficulties in linear algebra that students encountered. Also Dubinsky (1997) offers some thoughts about learning linear algebra.

What can we do to understand how our students learn? I interviewed with the education researches for advice at Yüzüncü Yıl University. They said shortly that teachers should listen carefully to their students. Also they suggested that teachers select a few students and interview them at regular intervals. It takes time to observe and listen to students what they are doing when they thinks about linear algebra. But, we will explore what our students thinking behind what they say. Keeping notes at the interviews can be useful. Than, these interviews may reveal some indications in the thinking and misconceptions of our students that lead us to better understanding of how they learn linear algebra.

5. Teaching Strategies

In order to enrich our teaching, we use from some teaching strategies. In linear algebra lectures, there are various alternatives, including using of projects, group activities, portfolios and technology.

Harel (2000) presents three 'principles' for the teaching of linear algebra, inspired by Piaget's psychological theory of concept development: the Concreteness Principle, the Necessity Principle and the Generalisability Principle. The Concreteness Principle states, "For students to abstract a mathematical structure from a given model of that structure, the elements of that model must be conceptual entities in the student's eyes". The Necessity Principle states, "Students must see an need to learn for what they are intended to be taught". The last, Generalisability Principle states, "When instruction is concerned with a 'concrete' model, that is a model that satisfies the Concreteness Principle, the instructional activities within this model should allow and encourage students to the generalisability of concepts".

I tried a few variations on "Mazur's polling method" (1996) in the linear algebra lectures. I used polling spontaneously and very quickly, to liven up lectures; Show hands. Who thinks that idea will work? Who doesn't? Who can make a conjecture? Is that possible or not? And so on. When students missed the following question, I used this method more formally. "If $\det(A) \neq 0$ and $\det(B) \neq 0$ (A and B are square matrices) then $\det(A + B) \neq 0$". I asked the class to vote on this question that 'true or false'. About half of them said true, half false. Then I asked students to find someone who disagreed with them and discuss which answer is really correct. I moved around the classroom, listening and occasionally asking pointed questions, for about 5 minutes. Then they voted again and about 85% got the correct answer. Some of the students who understood correctly explained how they went about answering it. This exercise took about 15 minutes, but it was a productive way to get students to think about how to analyze such a question, and to see how effective it can be to look for really simple examples.

6. Using Technology in Linear Algebra

Linear algebra teachers and mathematics education researches are agree that using technology is necessary at teaching and learning linear algebra. But it is a question that when, and how linear algebra teachers use technology (Herrero, 2000). There are several different roles that technology can play instruction, from eliminating computational drudgery in applications, to providing environments for actively exploring the properties of mathematical concepts and structures. Usually linear algebra teachers and authors prefer Matlab, Maple, Mathemtica, Mathwright, Cabri and Linalg. Some of teachers prefer computer projects to be done outside of class. Some of teachers use computer demos and examples to enrich lectures and to make a helpful visualization. Others rarely lecture at all with a significant proportion of class time spent interacting with the computer.

Computer programs provide students a means of instantly and effortlessly performing in linear algebra, and thus free them to concentrate on what the computations mean, and when and why to perform them. Many teachers use software in this context. The focus is not necessarily on linear algebra applications. Rather, students are intended to answer questions about what happens when certain computations are performed, without having to think too much about the mechanics of carrying out the operations. For example, students might experiment with the effect of triangular matrices as multipliers, without actually performing all the matrix multiplications by hand. There are two classic approaches about using computers in linear algebra course; first, some instructors think that doing some of the matrix multiplications by hand provides insight about why results appear as they do. Second, some instructors think that the ability to rapidly investigate a large number of examples makes a contribution to understanding.

Herero (2000)uses computer projects in linear algebra. The purpose of each of these projects is to introduce students to a new subject in linear algebra through a hands-on approach. They are intended to provide motivation for new definitions, show the need for the new theorems, make conjectures, and realize the usefulness of the new theorems by applying them to solve various problems. They can be used with any linear algebra software. Each student is allowed to choose an applied problem in the student's area of interest. The written project can be combined with an oral presentation. There are many good sources of problems, since many of the current linear algebra textbooks emphasize applications to a wide choice of different fields ((Bretscher, 1997; Cullen, 1996; Larson and Edvards, 1996; Lay,1997; Uhlig, 2000).

7. Conclusions

Mathematics education studies cannot give a certain solution to over-come all the difficulties in learning and teaching linear algebra. Many works have consisted in analysis of students' difficulties, trying of teaching strategies and experimental teaching, offering local solutions. Improving the teaching and learning of mathematics cannot consist in one remediation valid for all questions. It is a deeper knowledge of the nature of the concepts that help teachers makes their teaching better, richer and more expert.

Every linear algebra teacher knows how teach linear algebra in the sense that they all have plans or schedules what they will do the next courses. But they all need to understand better how students learn, and to recognize that the appropriate methods and context will be useful. Of course, there is no one right way to teach linear algebra. I hope that this article has had a good effect on the reader, and that the references may provide resources for further study.

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